INSTITUTE OF ENGINEERING AND TECHNOLOGY 610

NH-58, Delhi-Roorkee Highway, Baghpat Road, Meerut – 250 005 U.P. 12/6/23

Sessional Examination – II (SET A) : EVEN Semester 2022-23

Course/Branch: B Tech –I Year **(EP-1 TO EP-14)**

Subject Name : Engg. Mathematics-II **Semester: II**

Subject Code : BAS-203 **Max. Marks :60**

CO-3 *: Apply the concept of convergence in sequence, series and expansion of the function for Fourier series.*

CO-4 *: Apply the working methods of complex functions to find analytic functions*

Section – A (CO - 3) # Attempt both the questions # 30 Marks

Q.1 : Attempt any **SIX** questions (Short Answer Type). Each question is of two marks. (2 x 6 = 12 Marks)

a)	Discuss the convergence of following sequences. $a_n = \frac{n+1}{n}$	(BKL:K2 Level)
b)	Test the convergence of following series $2 + 2^2 + 2^3 + 2^4 + \cdots$	(BKL:K2 Level)
c)	Define Monotonic sequence with example.	(BKL:K1 Level)
d)	Write Dirichlet's conditions of Fourier series.	(BKL:K2 Level)
e)	If $f(x) = 1$ is expanded in a Fourier series in (0,2) then find the value of. a_n	(BKL:K2 Level)
f)	Find b_1 if the function $f(x) = x + x^2$ is expanded in Fourier series in $(-1,1)$.	(BKL:K2 Level)
g)	Find the value of the Fourier coefficient a_0 for the function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$.	(BKL:K2 Level)

Q.2 : Attempt any **THREE** questions (Medium Answer Type). Each question is of 6 marks. (3 x 6 = 18 Marks)

a)	Test the series: $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} \cdots$	(BKL:K3 Level)
b)	Test the series: $\frac{x}{1} + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1.3}{2.4} \cdot \frac{x^5}{5.} + \frac{1.3.5}{2.4.6} \cdot \frac{x^7}{7} + \cdots$	(BKL:K3 Level)
c)	Obtain the Fourier expansion of $f(x) = x \sin x$, as cosine series in $(0, \pi)$ and hence show that . $\frac{1}{1\times3} - \frac{1}{3\times5} + \frac{1}{5\times7} - \frac{1}{7\times9} + \cdots = \left(\frac{\pi-2}{4}\right)$	(BKL:K3 Level)
d)	Obtain the Fourier series for the function $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$ and hence, show that $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots = \frac{\pi^2}{12}$.	(BKL:K3 Level)
e)	Find Fourier Series for the function $f(x) = \begin{cases} x, 0 < x < 1 \\ 1 - x, 1 < x < 2 \end{cases}$	(BKL:K3 Level)

Section – B (CO - 4) # Attempt both the questions # 30 Marks

Q.3 : Attempt any **SIX** questions (Short Answer Type). Each question is of two marks. (2 x 6 = 12 Marks)

a)	State the necessary and sufficient conditions for a function $f(z)$ of a complex variable z, to be analytic in a region.	(BKL:K1 Level)
b)	Determine p such that the function $f(z)=\frac{1}{2}\log(x^2+y^2)+i\tan^{-1}\left(\frac{px}{y}\right)$ is an analytic function. Also find $f'(z)$.	(BKL:K2 Level)
c)	Define analytic function with an example.	(BKL:K1 Level)
d)	Show that complex function $f(z)=z^3$ is analytic.	(BKL:K2 Level)
e)	Define the harmonic function.	(BKL:K1 Level)
f)	If $u(x,y)=x^2-y^2$, prove that u satisfies Laplace equation.	(BKL:K2 Level)
g)	The function $f(x)=e^x(\cos y+i\sin y)$ is analytic or not.	(BKL:K2 Level)

Q.4 : Attempt any **THREE** questions (Medium Answer Type). Each question is of 6 marks. (3 x 6 = 18 Marks)

a)	Prove that the function $f(z)$ is defined by $f(z)=\frac{x^2y^5(x+iy)}{x^4+y^{10}}$, $z\neq 0$ and $f(0)=0$ satisfy Cauchy-Riemann equation at the origin, yet $f'(0)$ does not exist.	(BKL:K3 Level)
b)	If $f(z)=\begin{cases}\frac{x^3y(y-ix)}{x^6+y^2}; & \text{when } z\neq 0\\ 0; & \text{when } z=0\end{cases}$ Prove that $\frac{f(z)-f(0)}{z}\rightarrow 0$ as $z\rightarrow 0$ along any radius vector but not as $z\rightarrow 0$ in any manner.	(BKL:K3 Level)
c)	Find the analytic function whose imaginary part is $e^{-x}(x\cos y+y\sin y)$.	(BKL:K3 Level)
d)	Show that the function $u=\frac{1}{2}\log(x^2+y^2)$ is harmonic. Find the harmonic conjugate of u.	(BKL:K3 Level)
e)	If $u-v=e^x(x\cos x-y\sin y)$ and $f(z)=u+iv$ is analytic function. Find $f(z)$ in terms of z.	(BKL:K3 Level)

Roll No. : ....

MEERUT INSTITUTE OF ENGINEERING AND TECHNOLOGY

NH-58, Delhi-Roorkee Highway, Baghpat Road, Meerut – 250 005 U.P. 8/5/23

Sessional Examination – I (SET A) : EVEN Semester 2022-23

Course/Branch: B Tech –I Years **(EP-1 TO EP-14)** **Semester: II**

Subject Name : Engg. Mathematics-II **Max. Marks :60**

Subject Code : BAS-203

CO-1 : Apply the mathematical concepts for solving differential equations.

CO-2 : Apply the concept of Laplace Transform to solve differential equations

Section – A (CO - 1) # Attempt both the questions # 30 Marks

Q.1 : Attempt any **SIX** questions (Short Answer Type). Each question is of two marks. (2 x 6 = 12 Marks)

a)	Find the order and degree of the following differential equation $\frac{d^2y}{dx^2}+\sqrt{1+\left(\frac{dy}{dx}\right)^3}=0.$	(BKL:K2 Level)
b)	Find the P.I. of $\frac{d^2y}{dx^2}-y=x^2$.	(BKL:K2 Level)
c)	Solve $\frac{d^2y}{dx^2}+4y=\sin^2 2x$	(BKL:K2 Level)
d)	Solve $(D+1)^3y=2e^{-x}$	(BKL:K2 Level)
e)	Find the value of $\frac{1}{(D+3)}e^{-3x}\cos(x)$	(BKL:K2 Level)
f)	Find C.F of $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-12y=0.$	(BKL:K2 Level)
g)	Solve the simultaneous equations $\frac{dx}{dt}=3y, \frac{dy}{dt}=3x.$	(BKL:K1 Level)

Q.2 : Attempt any **THREE** questions (Medium Answer Type). Each question is of 6 marks. (3 x 6 = 18 Marks)

a)	Solve $\frac{d^2x}{dt^2}+\frac{dy}{dt}+3x=e^{-t}, \frac{d^2y}{dt^2}-4\frac{dx}{dt}+3y=\sin 2t$	(BKL:K3 Level)
b)	Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x\sin(\log x)$	(BKL:K3 Level)
c)	Solve the following differential equation by changing the independent variable $x\frac{d^2y}{dx^2}+(4x^2-1)\frac{dy}{dx}+4x^3y=2x^3.$	(BKL:K3 Level)
d)	Apply method of variation of parameters to solve: $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x.$	(BKL:K3 Level)
e)	An inductance L of 2.0 H and a resistance R of 20 Ω are connected in series with an e.m.f. of E volt. If the current i is zero when $t=0$, find the current i at the end of 0.01 second if $E=100\ V$, using the following differential equation $L\frac{di}{dt}+Ri=E.$	(BKL:K3 Level)

$-(1+D)(1-D)^{-1}$

Section – B (CO - 2) # Attempt both the questions # 30 Marks

Q.3 : Attempt any SIX questions (Short Answer Type). Each question is of two marks. (2 x 6 = 12 Marks)

a)	Explain first shifting property of Laplace transform.	(BKL:K2 Level)
b)	Find $L(t^3 e^{-3t})$.	(BKL:K2 Level)
c)	Find the Laplace Transform of $e^t\{1+u(t-3)\}$	(BKL:K2 Level)
d)	Find Laplace Transform of $\int_0^t \int_0^t \cos au\, du\, du$	(BKL:K2 Level)
e)	Evaluate: $L^{-1}\left(\frac{pe^{-2p}}{p^2-1}\right)$	(BKL:K2 Level)
f)	Find the Inverse Laplace Transform of $\log\left(1+\frac{1}{p^2}\right)$	(BKL:K2 Level)
g)	Find the Inverse Laplace Transform of $\frac{4p+8}{p^2+4p+5}$	(BKL:K2 Level)

Q.4 : Attempt any THREE questions (Medium Answer Type). Each question is of 6 marks. (3 x 6 = 18 Marks)

a)	Find inverse Laplace Transform of $\frac{p^2+2p+3}{(p^2+2p+2)(p^2+2p+5)}$	(BKL:K3 Level)
b)	Prove that $\int_{t=0}^{\infty}\int_{u=0}^{t} e^{-t}\frac{\sin u}{u}\,du\,dt=\frac{\pi}{4}$.	(BKL:K3 Level)
c)	Find the Laplace Transform of the following function of period 2a: $f(t)\begin{cases}\frac{h}{a}t, & 0<t<a\\ \frac{h}{a}(2a-t), & a<t<2a\end{cases}$	(BKL:K3 Level)
d)	State Convolution theorem and hence find inverse of Laplace Transform of following function $F(p)=\left\{\frac{p^2}{(p^2+4)^2}\right\}$	(BKL:K3 Level)
e)	Solve the following simultaneous DE's by using Laplace transform $\frac{dx}{dt}+4\frac{dy}{dt}-y=0, \frac{dx}{dt}+2y=e^{-t}$ With conditions $x(0)=y(0)=0$.	(BKL:K3 Level)

Roll No. : ....

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MEERUT INSTITUTE OF ENGINEERING AND TECHNOLOGY

NH-58, Delhi-Roorkee Highway, Baghpat Road, Meerut – 250 005 U.P.

Pre University Test (PUT) : Even Semester 2022-23

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Course/Branch	: B Tech –Ist Year		**Semester**	: II
Subject Name	: Engineering Maths-II	(EP-1 to EP-14).	**Max. Marks**	: 100
Subject Code	: BAS-203 (SET-B)		**Time**	: 180 min

CO-1 : Apply the mathematical concepts for solving differential equations.
CO-2 : Apply the concept of Laplace Transform to solve differential equations
CO-3 : Apply the concept of convergence in sequence, series and expansion of the function by Fourier series.
CO-4 : Apply the working methods of complex functions to find analytic functions.
CO-5 : Apply the concept of Taylor's series and Laurent's series for complex function and evaluation of integrals.

Section – A # 20 Marks (Short Answer Type Questions)

Attempt **ALL** the questions. Each Question is of 2 marks **(10 x 2 = 20 marks)**

Q.NO.		COx	Question Description # Attempt ALL the questions. Each Question is of 2 marks
1	A	CO1	The degree and order of the differential equation $\left(\frac{d^3y}{dx^3}\right)^4 - 6x^2\left(\frac{dy}{dx}\right)^8 = 0$ are ... and ...
	B	CO1	Find the Particular Integral of $(D^2 - 2D + 1)y = e^x x^3$.
	C	CO2	Find Laplace Transform of $F(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a \end{cases}$.
	D	CO2	Find Laplace Transform of $cosht.\sin 2t$.
	E	CO3	Discuss convergence of $\{1, 2, 2^2, 2^3, \ldots\}$
	F	CO3	Find Fourier coefficient a_n for $f(x) = xcosx$ in $-\pi < x < \pi$.
	G	CO4	Find Invariant points of transformation $w = \frac{1+z}{1-z}$
	H	CO4	Define conformal mapping.
	I	CO5	Evaluate $\int_{\lvert z\rvert=\frac{1}{2}} \frac{e^z}{z^2+1} dz$.
	J	CO5	Define singular point an analytic function. Find nature and location of the singularity of $f(z) = \frac{z - \sin z}{z^2}$.

Section – B # 30 Marks (Long / Medium Answer Type Questions)

Attempt **ALL** the questions. Each Question is of 6 marks **(5 x 6 = 30 marks)**

Q.2 (CO-1)	Solve the simultaneous equations $\frac{dx}{dt} + 5x - 2y = t, \frac{dy}{dt} + 2x + y = 0$ being given $x = 0, y = 0$ when $t = 0$. **OR** Solve by changing the independent variable $\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$.
Q.3 (CO-2)	Find the Laplace Transform of the triangular wave function of period 2c given by $f(t) = \begin{cases} t, & 0 < t \le c \\ 2c - t, & c < t < 2c \end{cases}$ **OR** Using Laplace Transform, solve initial value problem $\frac{d^2y}{dt^2} + 9y = 6\cos 3t\,;\, y(0) = 0\,,\, y'(0) = 0$
Q.4 (CO-3)	Test convergence: $1 + \frac{x}{2} + \frac{1.3}{2.4}x^2 + \frac{1.3.5}{2.4.6}x^3 + \cdots$ **OR** Find the half-range sine series of $f(x) = (lx - x^2)$ in the interval $(0, l)$. Hence, deduce that $\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$.
Q.5 (CO-4)	Prove that the function $f(z)$ defined by $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}; & z \ne 0 \\ 0; & z = 0 \end{cases}$ satisfies C.R. equations at the origin, yet $f'(0)$ does not exist. **OR** Show that $u(x, y) = x^3 - 4xy - 3xy^2$ is harmonic. Find its harmonic conjugate $v(x, y)$ and the corresponding analytic function $f(z) = u + iv$.

$\frac{1}{p^2 - a^2}$

Q.6 (CO-5)	Verify Cauchy's theorem for the function $f(z) = 3z^2 + iz - 4$ along the perimeter of the square with vertices $1 \pm i, -1 \pm i$. **OR** State Cauchy Integral formula. Hence evaluate $\int_C \frac{1}{(z^2+4)^2} dz$ where C is the circle $\vert z - i\vert = 2$.

Section – C # 50 Marks (Medium / Long Answer Type Questions)

Attempt **ALL** the questions. Each Question is of 10 marks.

Q.7 (CO-1)	Attempt any **TWO** questions. Each question is of **5** marks.
a.	Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8(e^{2x} + \sin 2x + x^2)$
b.	An R-L circuit has an e.m.f. given(in volts) by $4 \sin t$, a resistance of 100 ohms, an inductance of 4 henries with no initial current, Find the current at any time t.
c.	Use the variation of parameter method to solve the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$.

Q.8 (CO-2)	Attempt any **TWO** questions. Each question is of **5** marks.
a.	Find Laplace Transform of $e^{-4t}\int_0^t \frac{1-\cos 2t}{t} dt$
a.	Find the inverse Laplace Transform of $\log\left(\frac{p^2+4p+5}{p^2+2p+5}\right)$..
b.	State Convolution Theorem and hence evaluate $L^{-1}\left\{\frac{p}{(p^2+1)(p^2+4)}\right\}$.

Q.9 (CO-3)	Attempt any **ONE** question. Each question is of **10** marks.
a.	Obtain the Fourier series for the function $f(x) = \begin{cases} x, & -\pi < x \le 0 \\ -x, & 0 < x < \pi \end{cases}$ and hence, show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.
b.	Find Fourier Series for function $f(x) = \begin{cases} x, & 0 < x < 1 \\ 1 - x, & 1 < x < 2 \end{cases}$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

Q.10 (CO-4)	Attempt any **TWO** questions. Each question is of **5** marks.
a.	Find Mobius transformation that maps points $z = 0, -i, 2i$ into the points $w = 5i, \infty, -\frac{i}{3}$ respectively.
b.	F ind the analytic function $f(z) = u + iv$, if $u - v = (x - y)(x^2 + 4xy + y^2)$.
c.	Show that $f(z) = \log z$ is analytic everywhere in the complex plane except at the origin and its derivative is $\frac{1}{z}$

Q.11 (CO-5)	Attempt any **ONE** question. Each question is of **10** marks.
a.	Expand $\frac{1}{(z^2+4z+3)}$ in the regions: (i) $\vert z\vert < 1$, (ii) $1 < \vert z\vert < 3$, (iii) $\vert z\vert > 3$, (iv) $1 < \vert z + 1\vert < 2$.
b.	State Cauchy's Residue Theorem. Determine the poles and residues at each pole for: $\frac{z-1}{(z+1)^2(z-2)}$ and hence evaluate $\oint_C f(z)dz$, where C is the circle $\vert z - i\vert = 2$

Laplace $\frac{p^2}{2}$ -